

Group velocity



Group velocity is defined as the velocity in which the wave packet travels in a dispersive medium. Usually the wave group has the maximum amplitude at a particular value of 'x' and the velocity of this maximum amplitude point is known as the group velocity.

Therefore, the velocity at which a wave group (or a pulse) travels is the group velocity of the wave group. This velocity also signifies the velocity in which energy of the wave group is transmitted.

Expression for group velocity:

Let us consider a wave packet which has a group of two waves slightly different in angular frequencies and phase velocities but of equal amplitude for determining the expression for group velocity of a wave packet.

When their angular frequency are ω_1 and ω_2 Propagation Constants are k_1 and k_2 and the amplitude is A Then their separate displacement at any instant t can be represented as:



$$y_1 = a \sin(\omega_1 t - k_1 x) \quad \text{--- (1)}$$

$$\text{and } y_2 = a \sin(\omega_2 t - k_2 x) \quad \text{--- (2)}$$

From Young's Principle of Superposition, the resultant displacement y at any instant t is specified as

$$y = y_1 + y_2 = a \sin(\omega_1 t - k_1 x) + a \sin(\omega_2 t - k_2 x)$$

By using trigonometric relation, we get

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}, \text{ we get}$$

$$y = 2a \sin \left[\frac{(\omega_1 + \omega_2)t}{2} - \frac{(k_1 + k_2)x}{2} \right] \cos \left[\frac{(\omega_1 - \omega_2)t}{2} - \frac{(k_1 - k_2)x}{2} \right]$$

or

$$y = 2a \cos \left[\frac{(\omega_1 - \omega_2)t}{2} - \frac{(k_1 - k_2)x}{2} \right] \sin \left[\frac{(\omega_1 + \omega_2)t}{2} - \frac{(k_1 + k_2)x}{2} \right] \quad \text{--- (3)}$$

$$\underline{\text{or}} \quad y = A \sin \left[\frac{(\omega_1 + \omega_2)t}{2} - \frac{(k_1 + k_2)x}{2} \right] \quad \text{--- (4)}$$

$$\text{Here } A = 2a \cos \left[\frac{(\omega_1 - \omega_2)t}{2} - \frac{(k_1 - k_2)x}{2} \right]$$

Now, A is the modified amplitude of the wave packet. This modified amplitude of the wave packet is modulated in space and time both with a help of a very slowly varying envelope of frequency $(\omega_1 - \omega_2)/2$ and Propagation constant $(k_1 - k_2)/2$.

Its maximum value is $2a$.

VKSU equⁿ (3) can also be written as:

$$y = 2a \cos \left[\frac{\Delta\omega}{2} t - \frac{\Delta k}{2} x \right] \sin(\omega t - kx) \quad \text{--- (5)}$$

where,

$$\omega_1 - \omega_2 = \Delta\omega, \quad k_1 - k_2 = \Delta k,$$

$$\omega = \frac{\omega_1 + \omega_2}{2} \quad \text{and} \quad k = \frac{k_1 + k_2}{2}.$$

The group velocity V_g is the velocity in which this wave packet is represented by,

$$V_g = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \frac{\Delta\omega}{\Delta k} \quad \text{--- (6)}$$

when a bunch of waves has a number of frequency components in an infinitely small frequency interval then equⁿ (6) can be again written as:

$$V_g = \frac{d\omega}{dk} \quad \text{--- (7)}$$

Equⁿ (7) is equation of group velocity. This group velocity is also known as particle velocity.

Ex: when the frequency of matter wave is

$$\frac{1}{h} \left(\frac{1}{2} mv^2 \right) \quad \text{and the wavelength is } \frac{h}{mv},$$

find the group velocity.

Sol: Given,

$$\nu = \frac{1}{h} \left(\frac{1}{2} mv^2 \right) \quad \text{and} \quad \lambda = \frac{h}{mv}$$

we know that, group velocity

$$(V_g) = \frac{\omega}{k} = \frac{2\pi n\lambda}{2\pi} = n\lambda \quad \text{--- (1)}$$

Putting the value of 'n' and λ in eqn (1) we get

$$V_g = \frac{1}{h} \left[\frac{1}{2} mv^2 \right] \times \frac{h}{mv} = \underline{\underline{\frac{v}{2}}}$$